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Mach's Principle. Part 2. Realization of Dirac's Hypothesis in Brans–Dicke Theory with Cosmological Term

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Brans-Dicke theory supplemented with the scalar field potential of the form $m^6/\varphi \equiv Gm^6$ enables one to realize Dirac's "big numbers" hypothesis.

Eddington (1931a, b) was the first who noticed the approximate coincidence of the "big numbers" of microphysics and cosmology:

$$Gm^2 \approx \frac{H}{m} \tag{1}$$

$$G\varepsilon \approx H^2 \tag{2}$$

here G is the gravitational constant, m is a hadronic mass, H is Hubble's constant, $\varepsilon = n \cdot m$, n is the average baryon density in the universe. (Through out the paper we use units for which $\hbar = c = 1$.) It follows also from (1), (2) that

$$\frac{M_{\text{obs}}}{m} \approx \left(\frac{1}{Gm^2}\right)^2 \tag{3}$$

(the left-hand side is about 10^{80}, while $Gm^2 \approx 10^{-40}$), where M_{obs} is the observable mass of the matter in the universe.

Relation (2) holds in some cosmological models, in particular, for Friedmann's universe. Meanwhile (1) has no theoretical explanation till now. [For more details about the "big numbers" problem, see the books by Misner, Thorne, and Wheeler (1973), Weinberg (1972), Zeldovich and Novikov (1975).] In accordance with Dirac's (1937, 1938, 1973) hypothesis relation (1) should be an identity in cosmological time t, so G should depend on t. The well-known attempt to realize Dirac's hypothesis was the

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Brans–Dicke theory (Brans and Dicke, 1961, 1962). However "The mysterious relation (1) is not explained at all by the Brans–Dicke theory," as in this theory the t dependences of G and H are different and "the relation (1) can only be valid for a brief period in the history of the universe" [Cited from Weinberg's book (1972), p. 627]. Bekenstein and Meisel (1980) considered the problem in various generalizations of the Brans–Dicke theory. The author (Altshuler, 1968, 1969) tried to get (1) from quantum considerations, but did not succeed.

In the present paper it is shown that Brans–Dicke theory, supplemented with the scalar field φ potential of the form

$$V(\varphi)=\lambda m^6/\varphi \equiv \lambda G m^6 \tag{4}$$

(λ is a dimensionless constant), possesses necessary cosmological solutions, i.e., solutions with $G(t)\sim H(t)\sim 1/t$ and (1) is an identity.

Let the action be

$$S=\int\left\{-\varphi R+\omega\frac{1}{\varphi}\frac{\partial\varphi}{\partial x^k}\frac{\partial\varphi}{\partial x_k}-V(\varphi)+16\pi L^{(m)}\right\}(-g)^{1/2}\,d^4x \tag{5}$$

where ω is the Brans–Dicke dimensionless constant [post-Newtonian experiments demand $|\omega|>30$, Shapiro, Counselman and Ming (1976)]; $L^{(m)}$ is the matter Lagrangian. (We use here the notations of our preceding paper, I.) The action of the form (5) is often considered in view of the derivation of Einstein's gravity as the effect of spontaneously broken scale or grand-unification symmetries (Fujii, 1974; Minkowski, 1977; Matzuki, 1978; Linde, 1979; Smolin, 1979; Zee, 1979, 1980). The Brans–Dicke field is identified then with the Higgs field squared and in this case $L^{(m)}$ in (5) depends on φ. In this approach the potential $V(\varphi)$ is assumed to have a standard form for spontaneous breaking, i.e., has a minimum at $\varphi=\varphi_0\neq 0$. After the broken vacuum is established the theory is practically indistinguishable from conventional general relativity with gravitational constant $G=1/\varphi_0=\text{const}$.

The picture is changed essentially if $V(\varphi)$ is chosen in the form (4). Such a theory has no stable vacuum and perhaps it is not a defect but a virtue, and the time evolution of this nonstationary vacuum is just an evolution of our nonstationary universe. If so, in grand-unification theories only one phase transition is sufficient [e.g., at once $SU(5)\to SU(3)\otimes U(1)$], with the baryon-lepton and electroweak breakdown masses of the same order of magnitude. The modern "hierarchy" of these mass scales is a result of the time evolution of the universe in accordance with Dirac's hypothesis. [The possibility of the cosmological origin of mass scales was considered also by Terazawa (1981a).] Why the mass scales mentioned above have different time dependences is really the question of the origin of the action

(5), and it is beyond the scope of the present work. However, some speculations about a possible origin of the Brans–Dicke theory will be given in conclusion.

Variation of the action (5) over the fields g_{ik}, φ gives the following dynamical equations:

$$\frac{\delta}{\delta g^{ik}}:\ \varphi(R_{ik}-\tfrac{1}{2}g_{ik}R)=8\pi T_{ik}^{(m)}+\varphi_{;ik}-g_{ik}\Box\varphi$$

$$+\frac{\omega}{\varphi}\left[\frac{\partial\varphi}{\partial x^i}\frac{\partial\varphi}{\partial x^k}-\frac{1}{2}g_{ik}\frac{\partial\varphi}{\partial x^n}\frac{\partial\varphi}{\partial x_n}\right]+\tfrac{1}{2}g_{ik}V \qquad (6)$$

$$\frac{\delta}{\delta\varphi}:\ -R-\frac{2\omega}{\varphi}\Box\varphi+\frac{\omega}{\varphi^2}\frac{\partial\varphi}{\partial x^n}\frac{\partial\varphi}{\partial x_n}-\frac{\partial V}{\partial\varphi}+16\pi\frac{\delta L^{(m)}}{\delta\varphi}=0 \qquad (7)$$

And substituting the trace of (6) into (7) one obtains

$$\Box\varphi=\frac{1}{(3+2\omega)}\left[8\pi T^{(m)}+2V-\varphi\frac{\partial V}{\partial\varphi}+16\pi\varphi\frac{\delta L^{(m)}}{\delta\varphi}\right] \qquad (8)$$

($T_{ik}^{(m)}$ is the matter energy-momentum tensor; $T^{(m)}$ its trace.)

Let us study the simplest case of spatially homogeneous and isotropic Friedmann–Robertson–Walker world with space-time interval

$$ds^2=dt^2-a^2(t)\,d\Sigma_K^2, \qquad (9)$$

$K=+1, 0, -1$ for the closed, flat, and open models. The energy–momentum tensor is taken to be that of a perfect fluid

$$T_{ik}^{(m)}=(p+\varepsilon)U_iU_k-pg_{ik} \qquad (10)$$

(ε, p, and U_k are the energy density, pressure, and velocity four-vector, respectively) and $L^{(m)}$ in (5) is supposed to be independent of the scalar field φ, i.e.,

$$T^{(m)k}{}_{i;k}=0 \qquad (11)$$

For the metric (9) the 00 component of (6) and equations (8), (11) will become

$$\frac{\dot a^2}{a^2}+\frac{K}{a^2}=\frac{8\pi\varepsilon}{3\varphi}+\frac{\omega}{6}\frac{\dot\varphi^2}{\varphi^2}-\frac{\dot a}{a}\frac{\dot\varphi}{\varphi}+\frac{\lambda m^6}{6\varphi^2} \qquad (12)$$

$$\ddot\varphi+\frac{3\dot a}{a}\dot\varphi=\frac{1}{3+2\omega}\left[8\pi(\varepsilon-3p)+\frac{3\lambda m^6}{\varphi}\right] \qquad (13)$$

$$\dot\varepsilon+\frac{3\dot a}{a}(\varepsilon+p)=0 \qquad (14)$$

[Definitions (4), (10) are used; the dot means t derivative.]

It is well known that these equations should be supplied with the matter equation of state $p = p(\varepsilon)$.

Three particular solutions of (12)-(14) are given below.

I. Zero-curvature three space ($K = 0$); matter is absent ($\varepsilon = p = 0$). The particular solution of equations (12), (13) is

$$\varphi = \left[\frac{3\lambda}{\omega(3+2\omega)}\right]^{1/2} m^3 t, \qquad a = \text{const} \cdot t^{\omega/3} \tag{15}$$

It gives expansion for $\omega > 0$ (and therefore $\lambda > 0$). Solution (15) evidently does not exist in the classical Brans-Dicke theory, when $\lambda = 0$. Equation (15) gives for the Hubble's "constant"

$$H \equiv \frac{\dot{a}}{a} = \frac{\omega}{3}\frac{1}{t} \tag{16}$$

and relation (1) ($G \equiv 1/\varphi$) is fulfilled for $\lambda = 6 + 9/\omega$. However, this connection of λ and ω should not be taken too seriously, since relation (1) holds only by the order of magnitude. It is more important that (1) is an identity in the time variable and this result does not depend on the value of ω.

A simple analysis shows that solution (15) is stable. Taking variations of the fields $\varphi(t)$, $a(t)$ in the form

$$\varphi \to \varphi(1 + \xi(t)), \qquad a \to a(1 + \alpha(t)) \tag{17}$$

for small ξ, α, one gets a linear system [see below (22)]. Its general solution for the unperturbed fields (15) is

$$\xi(t) = \frac{C_1}{t} + \frac{C_2}{t^\omega}$$

$$\alpha(t) = \alpha_0 + \frac{\omega}{3}\frac{C_1}{t} + \frac{1}{3}\frac{C_2}{t^\omega}$$

C_1, C_2, α_0 are arbitrary constants; α_0 gives a trivial change of the constant in (15) and hence may be discarded. The decrease in ξ, α with time means the stability of the solution (15), at any rate with respect to spatially homogeneous variations.

Nonzero spatial curvature ($K = \pm 1$) does not change solution (15) substantially, because the term K/a^2 in (12) decreases with time faster than $1/t^2$ for $\omega > 3$. The same is true for matter; (14) gives an isentropic and very fast decrease of ε, p [for a from (15) and for any conventional matter equation of state]. Hence the ε, p terms in the right-hand side of (12), (13) are negligible. But then we get a direct contradiction to the relation (2), and in general to all observations. The problem is the same as in the stationary universe of Bondi and Gold (1948) and Hoyle (1948, 1949); but in a softened version since here $a(t)$ grows as a power of t, not as an exponential function. And in our case, like that of the stationary model, too fast decrease in ε must be compensated by the continuous creation of matter or, in other words, by the continuous transformation of the scalar field energy into the matter energy. In principle it is possible if $L^{(m)}$ in (5) depends on φ and matter continuity equation (11) is canceled. (For example, by production of baryons in galaxy nuclei, which in turn are formed because of a gravitational instability of spatially homogeneous scalar field energy distribution.) However, it is hardly possible that the observable galaxies distribution and the observable spectrum of the microwave radiation can be produced in such a manner (see about it in Weinberg, 1972).

II. Closed or open model ($K=\pm 1$); $\varepsilon = p = 0$. The particular "vacuum" solution of equations (12) and (13) is

$$\varphi = \left(\frac{\lambda}{3+2\omega}\right)^{1/2} m^3 t, \qquad a = \left(\frac{2K}{\omega-3}\right)^{1/2} \cdot t \tag{18}$$

This solution exists for the closed model ($K=1$) if $\omega > 0$, $\lambda > 0$ and for the open model ($K=-1$) if $\omega < 0$, $\lambda < 0$ (note that $|\omega| \gg 1$). The general solution of the linear system (22) for small variation (17) follows the power law

$$\xi \sim t^n, \qquad \alpha \sim t^n$$

where the power degree is found from the cubic equation

$$n^3 + 4n^2 + (9-2\omega)n + (6-2\omega) = 0$$

It is easy to show that for $\omega < 0$ all three roots have negative real parts, i.e., in this case all linear independent solutions of (22) decrease with time. Hence the open model (18) is stable. On the contrary, the closed model (18) (when $\omega > 0$) is unstable.

Relation (1) for the model (18) is fulfilled but (2) is evidently wrong as ε decreases too fast (e.g., $\varepsilon \sim t^{-3}$ for the "dust" matter). The continuous creation of matter should be supposed here as well as in the model (15).

III. In the more conventional and realistic approach the direct interaction of matter and scalar field may be essential only in the initial superdense stage of the Big Bang. Later the continuity equation (11) becomes true and in particular, for the "dust" matter we have $\varepsilon \sim a^{-3}$. In this case, as it was noted by Dirac (1937, 1938, 1973), the simultaneous validity of (1), (2) demands

$$\frac{1}{G} = \varphi \sim t, \qquad a \sim t^{1/3}, \qquad \varepsilon \sim \frac{1}{t} \tag{19}$$

It turns out that for the flat model ($K=0$) the system (12)–(14) possesses solution of the type (19) if constants ω, λ are negative. This particular solution for the general matter equation of state $p = \gamma\varepsilon$ ($\gamma = \text{const}$) is

$$\varphi = \left[\frac{3\lambda}{2+6\gamma+3\omega(1-\gamma^2)}\right]^{1/2}(1+\gamma)m^3 t$$
$$a = \text{const}\cdot t^{1/3(1+\gamma)} \tag{20}$$
$$\varepsilon = \frac{1-\omega(1+\gamma)}{8\pi(1+\gamma)^2}\cdot\frac{\varphi}{t^2}\sim\frac{1}{t}$$

Hence for $\gamma = 0$ the Dirac theory (19) follows:

$$\varphi = \left(\frac{3\lambda}{2+3\omega}\right)^{1/2} m^3 t, \qquad a = \text{const}\cdot t^{1/3}$$
$$\varepsilon = \frac{1-\omega}{8\pi}\left(\frac{3\lambda}{2+3\omega}\right)^{1/2}\frac{m^3}{t} \tag{21}$$

As is seen, the positiveness of the energy density ε demands $\omega < 0$ ($|\omega| \gg 1$) and therefore $\lambda < 0$. Let us show that in spite of the "wrong" sign ($\omega < 0$) of the scalar field kinetic term in the action (5) and in spite of the unboundedness of $V(\varphi)$ from below ($\lambda < 0$ means $V \to -\infty$ for $\varphi \to 0$) the solution (21) is stable with respect to small variations (17) of the fields $\varphi(t)$, $a(t)$ and the small energy density variation [$\varepsilon \to \varepsilon + \varepsilon_1(t)$]. Variation of equations (12)-(14) gives for d, ξ, ε_1 the following linear system ($p = 0$; $H = \dot{a}/a$; $\phi = \dot{\varphi}/\varphi$):

$$(2H+\phi)\dot{\alpha} - \frac{2K}{a^2}\alpha = \frac{8\pi\varepsilon_1}{3\varphi} + \left(\frac{\omega}{3}\phi - H\right)\dot{\xi} - \left(\frac{8\pi\varepsilon}{3\varphi} + \frac{\lambda m^6}{3\varphi^2}\right)\xi$$
$$\ddot{\xi} + (2\phi + 3H)\dot{\xi} + 3\phi\dot{\alpha} = \frac{1}{(3+2\omega)}\left[\frac{8\pi\varepsilon_1}{\varphi} - \left(\frac{8\pi\varepsilon}{\varphi} + \frac{6\lambda m^6}{\varphi^2}\right)\xi\right] \tag{22}$$
$$\dot{\varepsilon}_1 + 3H\varepsilon_1 + 3\varepsilon\dot{\alpha} = 0$$

This system was used above for investigation of stability properties of solutions (15), (18). For the unperturbed particular solution (21) the general solution of the system (22) has the form

$$\alpha \sim t^n, \qquad \xi \sim t^n, \qquad \varepsilon_1 \sim t^{n-1} \tag{23}$$

where power degree n should be found from the following cubic equation:

$$n^3 + 2n^2 - \frac{3\omega^2 - 3\omega - 5}{3+2\omega}n - \frac{3\omega^2 - \omega - 2}{3+2\omega} = 0 \tag{24}$$

For $\omega < -3/2$ the real parts of all the three roots are negative, therefore time evolution will inevitably lead to $\alpha \ll 1$, $\xi \ll 1$, $\varepsilon_1 \ll \varepsilon$ and hence the solution (21) is solved, in spite of $\omega < 0$, $\lambda < 0$.

We should note that the comparison of the Brans-Dicke post-Newtonian parameters (Misner *et al.*, 1973, *p.* 1072) with experiment does not say anything about the sign of ω. For $\omega < 0$ the contribution to the scalar field from the near-by matter is negative, but it does not mean that the gravitational constant is negative in general cosmological solution [see, e.g., (21)]. The Brans-Dicke theory with $\omega < 0$ was considered also by Miyazaki (1979). However, the question of consistency of the action (5) with $\omega < 0$, $\lambda < 0$ needs a further investigation, in particular in connection with Mach's principle (see preceding paper I).

It is easy to show that relation (3) is fulfilled in the model (21). Calculation of the observable mass of the universe by the familiar formula (Landau and Lifshitz, 1962)

$$M_{\rm obs} = 4\pi \int_0^{\eta} \varepsilon a^3 \chi^2 \, d\chi \tag{25}$$

(parameter η is defined by $dt = ad\eta$) gives for (21)

$$M_{\rm obs} = \frac{9}{16}(1-\omega)\left(\frac{3\lambda}{2+3\omega}\right)^{1/2} m^3 t^2 \tag{26}$$

From (26), (21) it follows that (3) holds for any time.

The whole consideration above could be carried out in another scale gauges, e.g., in the gauge of constant G and variable particle mass m. In this gauge solutions (15), (18), (21) will lead to m decreasing with time by the law $m \sim t^{-1/3}$.

What is the observational status of Dirac's theory (19) and therefore of the model (21)? The age of the universe $t_0 = 1/3H_0 \approx 6 \times 10^9$ years (H_0 is the observable Hubble constant, $H_0^{-1} \approx 1.8 \times 10$ years). The logarithmic time derivative of the gravitational constant is

$$\left|\frac{\dot{G}}{G}\right| = 3H_0 \approx 2 \times 10^{-10}\,(\text{yr})^{-1}.$$

These numbers are incompatible with experiment, or at any rate are on the edge of the allowable (Misner *et al.*, 1973; Weinberg, 1972; Zeldovich and Novikov, 1975). The model (18), where $a \sim t$, gives $|\dot{G}/G| = H_0 \approx 6 \times 10^{-11}(\text{yr})^{-1}$; it is not in contradiction with observations so far. In the solution (15) G varies even more slowly [$|\dot{G}/G = 3H_0/\omega$, cf. (16)]. But models (15), (18) have their own problems, as was already mentioned.

Perhaps the theory described by equations (4), (5) needs a modification. In this connection the question arises: what may be a more fundamental scheme that results in the action (5) in some phenomenological approximation? The following speculation about the possible origin of the Brans-Dicke theory out of the quantum principles probably may be useful.

In 1967 Sakharov (1967) proposed identifying Einstein's gravitational action with that of the quantum fluctuation of matter (cf. also Misner et al., 1973; Zeldovich and Novikov, 1975). This idea, called Pregeometry, was developed farther (Adler, 1980a, b; Akama, 1981; Terazawa, 1981b;). Sakharov (1975) showed that the quartic and quadratic ultraviolet divergences are absent in the effective quantum action and the gravitational constant is defined by the masses of elementary quantized fields. This means, e.g., that the elementary Fermi field Lagrangian $L_0 = \bar{\Psi}(\hat{\partial} - \hat{\Gamma} - m)\psi$ being considered as functional of two external fields $g_{ik}(x)$, $m(x)$ will give a scale-invariant quantum action for these fields:

$$S \sim \left[6 \frac{\partial m}{\partial X^k} \frac{\partial m}{\partial X_k} + m^2 R \right] \tag{27}$$

This result is necessarily obtained when the scale-invariant regularization method is used. In the scale gauge $m = \text{const}$ the unphysical version of Einstein's theory results with the Planck mass being equal to the elementary fermion mass.

However, the situation may be quite different if the elementary Lagrangian depends on two or more external scalar fields, a Lagrangian containing several Higgs fields or a neutrino Lagrangian with Dirac (m) and Majorana (M) masses. (The consideration is purely illustrative and hence is oversimplified.) In this case, besides the terms of the form (27) the quantum effective action for the fields $g_{ik}(x)$, $m(x)$, $M(x)$ should contain the scale-invariant kinetic terms of another form. Dimensional considerations suggest that the logarithmically divergent kinetic part of the action must become

$$S \sim \left(6 \frac{\partial m}{\partial X^k} \frac{\partial m}{\partial X_k} + m^2 R \right) + \left[6 \frac{\partial M}{\partial X^k} \frac{\partial M}{\partial X_k} + M^2 R \right] + \text{const} \cdot (m^2 + M^2) \frac{\partial \ln(M/m)}{\partial X^k} \frac{\partial \ln(M/m)}{\partial X_k}. \tag{28}$$

If $M \gg m$ (28) transforms into the action of the Brans-Dicke theory with the scalar field determined by the largest of the two masses ($\varphi \approx M^2$). To the author's knowledge, such a program of deduction of the Brans-Dicke theory has not been worked out so far. If the action is symmetrical in the mass field [as, e.g., (28) is], then the growing difference of the masses in the process of the cosmological evolution may be the result of instability of the $m \leftrightarrow M$ symmetric initial state, i.e., of some spontaneous breakdown of this symmetry back at the "quantum era" (when $M \approx m$).

Nothing was said until now about the origin of the potential (4). The cosmological term (vacuum energy) problem is one of the most difficult in modern theoretical physics. From the quantum point of view (4) is the finite correction to the potential terms in the quantum effective action (28) caused by the gravitational interaction of elementary fields (mass m) (Zeldovich, 1968), or, perhaps, by exchange of a superheavy gluon ($M_x^2 \approx \varphi$). In some future supergravity the potential (4) will probably be the first non vanishing term of quantum vacuum energy. The purpose of the present paper was to study the cosmological consequences of this possible fact.

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